Problem 4: Secret Santa (15 points) [CodeForces]

By Pranav Mathur

Problem Statement

This year in ACM, N club members, numbered from 1 to N, are participating in a secret Santa gift exchange! Before the exchange takes place, each member of the club is assigned exactly one other member to get a gift for, and no two members of the club are assigned the same person.

Unfortunately, while writing their solution for a CTF/Security-style problem for COSCON this year, one of the officers accidentally unleashed a script that deleted many of the Secret Santa assignments on their laptop, leaving us with only M of the original assignments! (Naturally, the CTF problem was dropped from the contest). After this disaster, Arya, an avid but struggling combinatorist, began thinking about the number of full Secret Santa assignments that satisfy the remaining assignments. Help him answer this question!

Input/Output Details

Input: The first line of input consists of two space-separated integers N and M, where N is the total number of members participating in the gift exchange, and $M \leq N$ is the number of gift assignments we are left with.

The next M lines input each contain two space-separated integers a and b, indicating that member a is assigned to give a gift to member b. Note that if a giving a gift to b, it is *possible but not necessarily true* that b is giving a gift to a. Furthermore, each number in $\{1, ..., N\}$ will appear at most once as a and at most once as b, since no one is giving to more than one person and no one is receiving a gift from more than one person.

Output: Print a single integer, the number of full Secret Santa assignments of the N members that satisfy the M given assignments. Because this number can be large, print it **modulo** $10^9 + 7$.

Constraints:

$2 \leq N \leq 1000$

$0 \leq M \leq N$

$1 \leq a, b \leq N$

Partial Credit: In some of the test files, it will be guaranteed that each member $\{1, ..., N\}$ appears at least once in the given assignments. Partial credit (5 pts) will be awarded for solutions that succeed on these test cases.

Sample Input 1

```
6 3
1 3
2 4
5 6
```

Sample Output 1

```
6
```

Sample 1 Explanation: We have $N = 6$ and $M = 3$. The possible full assignments are given below

- $1 \to 3$, $3 \to 1$, $2 \to 4$, $4 \to 2$, $5 \to 6$, $6 \to 5$.
- $1 \to 3$, $3 \to 1$, $2 \to 4$, $4 \to 5$, $5 \to 6$, $6 \to 2$.
- $1 \to 3$, $3 \to 2$, $2 \to 4$, $4 \to 5$, $5 \to 6$, $6 \to 1$.
- $1 \to 3$, $3 \to 2$, $2 \to 4$, $4 \to 1$, $5 \to 6$, $6 \to 5$.
- $1 \to 3$, $3 \to 5$, $2 \to 4$, $4 \to 1$, $5 \to 6$, $6 \to 2$.
- $1 \to 3$, $3 \to 5$, $2 \to 4$, $4 \to 2$, $5 \to 6$, $6 \to 1$.

Sample Input 2

```
7 4
2 3
3 5
4 7
7 4
```

Sample Output 2

```
3
```

Sample 2 Hint: observe that $1 \to 2$, $2 \to 3$, $3 \to 5$, $4 \to 7$, $5 \to 6$, $6 \to 1$, $7 \to 4$ is a valid full assignment, while $1 \to 2$, $2 \to 3$, $3 \to 5$, $4 \to 7$, $5 \to 1$, $6 \to 6$, $7 \to 4$ is not (why?).